C-JTT-M-TUD

STATISTICS – IV

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

Candidates should attempt FIVE questions in all including Questions No. 1 and 5 which are compulsory and attempt remaining THREE questions by choosing at least ONE each from Sections A and B.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written only in ENGLISH.

Symbols and abbreviations are as usual.

If any data is required to be assumed for answering a question, it may be suitably assumed, indicating this clearly.

Any graphs/sketches are to be drawn on the answer-book itself, wherever required.

All parts/sub-parts of a question being attempted must be completed before moving on to the next question.

SECTION A

1. (a) For an aperiodic, positive persistent state j, show that $\lim_{n \to \infty} p_{jj}^{(n)} = 1 / \mu_j$, where μ_j denotes the mean recurrence time of state j. *8*

(b) For a Poisson process $\{X(t), t \geq 0\}$, show that the conditional distribution of X(s) given X(t), $0 < s < t$ is binomial. *8*

(c) Describe the pure birth process. Derive an expression for $P_{0n}(0, t) = P\{X(t) = n \mid X(0) = 0\}$ for the Yule – Furry process. *8*

(d) Define (i) lead time (ii) buffer stock in an inventory problem. How do you determine the buffer stock in an EOQ situation ? *8*

(e) Explain the 'Gap test' and the 'Poker test' for testing randomness of observations from uniform distribution. *8*

2. (a) Let $\{\xi_n, n \geq 1\}$ be a sequence of i.i.d.r.v.'s with $P(\xi_k = 1) = p$ and $P(\xi_k = 0) = q$, $p + q = 1$.

Define $Y_n = n - \sum_{i=1}^{n} \xi_i$, $n = 1, 2, \ldots$.

Show that $\{Y_n, n \geq 1\}$ is a Markov chain. Obtain its one-step transition probability matrix. *10*

(b) The t.p.m. of a M.C. with S = {1, 2, 3} is given below :

$$P = \begin{bmatrix} 2/3 & 0 & 1/3 \\ 1/4 & 3/4 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

Obtain the invariant distribution of the chain. *10*

(c) Derive the difference-differential equations for the $M \mid M \mid c \mid \infty$ queueing system and obtain its solution in steady state conditions. *10*

(d) Define G.W. branching process. Let $\{X_n\}$ denote a G.W. branching process with $E(X_1) = m$ and $Var(X_1) = \sigma^2$. Derive $E(X_n)$ and $Var(X_n)$. *10*

3. (a) Derive a set of sufficient conditions to declare a given basic feasible solution to be an optimal solution to a L.P.P. *10*

(b) Explain the problem of duality in L.P.P. Prove that if the primal has an optimal solution, then there exists an optimal solution to the dual. *10*

(c) Derive the difference-differential equations for the finite capacity $M \mid M \mid 1$: FIFO $\mid N \mid \infty$ queue and obtain the queue length distribution in steady state conditions. Derive an expression for the mean queue length. *10*

(d) Derive the formula for the optimal order quantity for a single period deterministic inventory model assuming that the demand and replenishment both occur at a uniform rate. *10*

4. (a) In the classical ruin problem, show that when the adversary is infinitely rich, the probability of gambler's ruin x_k is given by

$$x_k = \begin{cases} \left(\frac{q}{p}\right)^k & \text{if } p > q \\ 1 & \text{if } p \leq q \end{cases}$$ *10*

(b) Describe the $M \mid M \mid c \mid \infty$ queueing system. Show that it is a particular case of birth and death process. *10*

(c) Solve the following L.P.P. by Big-M method :

$$\text{Maximize } Z = -x_1 + 3x_2$$

$$\text{subject to } \begin{aligned} x_1 + 2x_2 &\geq 2 \\ 3x_1 + x_2 &\leq 3 \\ x_1 &\leq 4 \\ x_1, x_2 &\geq 0. \end{aligned}$$ *10*

(d) Explain 'individual replacement policy' and 'group replacement policy'. When do you prefer a group replacement policy to an individual replacement policy ? *10*

SECTION B

5. (a) Discuss Chiang's method of constructing an abridged life table. *8*

(b) "Model life table is used to estimate basic demographic measures from incomplete data." Discuss. *8*

(c) Enumerate the factors influencing internal migration. Explain how internal migration affects the population composition of the sending and the receiving areas. *8*

(d) Explain the following : *8*

(i) Address Bus

(ii) Control Bus

(iii) Data Bus

(e) Describe memory systems for microcomputers. *8*

6. (a) Show that age structure of stable population does not depend on the initial age distribution of the population. *10*

(b) Discuss the properties of logistic curve. *10*

(c) Calculate the crude and standardised death rates for the local population from the following data and compare them with crude death rate of the standard population. *10*

Age-group	Standard population	Deaths	Local population	Deaths
0 – 10	600	18	400	16
10 – 20	1000	5	1500	6
20 – 60	3000	24	2400	24
60 – 100	400	20	700	21

(d) Explain Myer's index and discuss its uses in detecting digit preference. *10*

7. (a) Describe the main sources of demographic data in India and suggest measures to improve the quality of the data. *10*

(b) Describe two operating systems for personal computers. *10*

(c) Explain the following : *10*

(i) Bit density

(ii) Bit handling

(iii) Bit mapping

(iv) Bit rate

(d) Describe the hierarchical approach of database system. *10*

8. (a) Describe the matrix method of population projection. *10*

(b) Describe in brief the various stages and operational set-up of 2001 Census of India. *10*

(c) Explain the following : *10*

(i) Debugging

(ii) Bar coding

(iii) Indexed sequential file

(d) Describe a flow chart to find out the highest marks obtained in an examination and the roll number of the student obtaining the highest marks. *10*